MODELING AND ANALYSIS OF THE ECOMOMIC DEVELOPMENT OF AZERBAIJAN BASED ON THE PRODUCTION FUNCTION

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Abstract: The study aims to identify the economic development potential by modeling and identifying the turning points in the development of the Azerbaijani economy corresponding to the phases in the economic cycle. Overall, the paper presents the results of the analysis of the development of the Azerbaijani economy using production functions (PF). The empirical characteristics of the PF made it possible to periodize the considered time interval i.e. identify areas with different behavior of empirical characteristics. Moreover, it was revealed that these sections are repeated within a certain period. The specifics of economy modeling are determined using the methods of production variables based on the annual data of the State Statistical Committee for 1997–2021. The methodological basis of the study is the economic and mathematical apparatus of the classic Cobb–Douglas production function. The calculations were performed using freely distributed modern software, the statistical environment R: which is one of the most dynamically developing programs in its class. As a result of the study, we identified the chronological boundaries of short-term and medium-term economic cycles in the economy of Azerbaijan, analyzed the patterns of production variables in different periods of the cycle. The article provides the results of modeling the GDP dynamics in Azerbaijan, which are described by production functions with a significantly different set of parameters; identifies important macroeconomic variables; provides an econometric analysis of the models, production variables, and corresponding conclusions.

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Introduction

Production functions explaining the statistical relation between the GDP output and production inputs (aggregated indices of resources) belong to the well-known and widely used multiplicative economic and mathematical models. Production functions (PF) are mainly used for analysis, modeling, planning, and forecasting (Tsvetkov & Zoidov, 2010).

Cobb and Douglas (1928) was the pioneering study on the production function. This study offered mathematical specifications and empirical estimations of the relationship between output volume and production factors. Based on the analysis of patterns in the indices of fixed capital, labor force, and production, the study concluded that the output could be estimated by using the following formula:

$$y = f(x_1, x_2) = A*x_1^{\alpha} *x_2^{\beta} \qquad (1)$$

where x_1 is the capital index, x_2 is the labor index.

The Cobb–Douglas production function turned out to be an extremely convenient mathematical tool for describing production processes, making it one of the most popular production functions in applied economic research. Many forms and types of the production function were proposed later, like the widely used CES function (Plakunov & Rajackas, 1984), and the generalized Leontief function (Diewert, 1971), the generalized production function of Revankar and Zellner (1969), and others.

Griliches and Mairesse (1995) analysed the economic system at the micro and macro level based on the Cobb–Douglas production. There are many large-scale studies by Green (2016), Zellner, Kmenta, and Dreze (1966) were conducting on economies of scale using the production function. Hasanli (1998) worked on CES function and its derivatives to study the transition to multifactor models.

Many factors affect the production level, such as capital, optimal resource allocation, technology, innovations, etc. Therefore, in the present study, an empirical analysis of substitution elasticity, the direction of technical changes, and the degree of contribution of the factors of Azerbaijan's economic growth to the overall performance of production factors has been done.

An important point to note is that the mechanism of production functions is quite well elaborated for developed economies. The stable economies of the developed countries make it possible to obtain sufficient data of time series corresponding to a system with almost unchanged properties, which is appropriate for correct econometric analysis. In a transitional economy, the situation is substantially different. A sharp rise in the economic processes results in a much faster loss of comparability between neighboring members in the time series analysis compared to a stable economy (Bessonov & Tsukhlo, 2002).

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Another specific problem for transitional economies, like Azerbaijan, is related to the determination of the production functions. For example, it is difficult, challenging, and often impossible to provide accurate market estimates of factors of production during the transition period, characterized by relatively rapid structural changes. The fact is that during intensive structural changes in transitive and transitional economies, composite economic indicators used for estimating production functions can lack some essential information on the movement of the entire set of statistical data, which may give rise to ambiguity in the results.

Bessonov (2002) considered the issue of adequate reflection of the fixed asset assessment in transitive and transitional economies. This logically justifies the use of relative, logarithmic values instead of absolute values or an unconventional set of factors of production, such as investments instead of funds for the econometric analysis of a transitional economy. Model 10 is formulated accordingly.

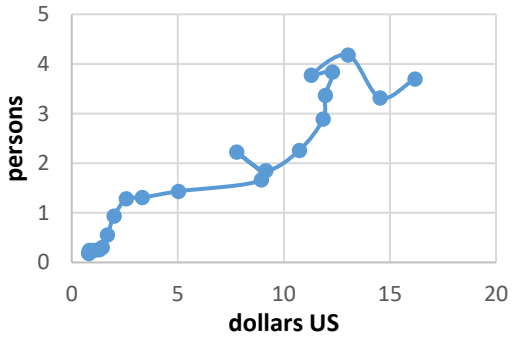
Thus, the study assumes the existence of a positive "balance" (unaccounted effect), i.e., there is an assumption that the capital (K) and labor resources (L) do not fully explain the dynamics of the output (Y) (GDP). Secondly, the statistics on fixed assets do not reflect their accurate assessment, which is expected in a dynamically developing transitive economy like Azerbaijan. It justifies the assumptions described above. The production factor—investments in fixed assets—meets all the necessary conditions (Bessonov & Tsukhlo, 2002).

Verification of proposed hypothesis

To check the basic premises, the following relative indicators are calculated based on the data presented in figures 1 and 2 (Y indicates the Gross Domestic Product (GDP) of Azerbaijan; I shows investments in fixed assets at comparable prices; L represents labor resources i.e. number of the employed population of the country)

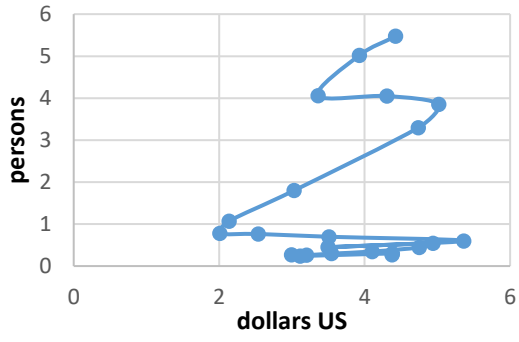
Figure 1, based on real data (adjusted for inflation), shows a graph of the dependence of average productivity l=Y/L on average capital/labor ratio k=K/L in Azerbaijan in 1997–2021. Figure 1 clearly illustrates that the average labor productivity increases with the increase in the average capital/labor ratio, and the production function curve y(k) is generally convex, as it should be for a linear homogeneous production function, expect during the economic crisis in Azerbaijan. Figure 2 highlights the turning points in 2001, 2005, 2009, and 2014, corresponding to the chronology of economic cycles identified in the economic development of Azerbaijan.

Figure 1: Effect of average labor productivity on average capital/labor ratio during 1997–2021 in Azerbaijan (Industry of Azerbaijan, 2021).



Source: Author

Figure 2: Effect of average capital productivity on average employment during 1997–2021 in Azerbaijan (Industry of Azerbaijan, 2021).



Source: Author

Figure 2 shows the effect of average capital productivity on average employment in Azerbaijan. The convex shape of the curves y(k) and y(l) is consistent with the law of diminishing returns, which confirms the assumptions indicated above. The direction of the convexity changed in 1999, 2001, 2005, 2009, and 2015.

Figures 1 and 2 illustrate the turning points in the economic development of Azerbaijan, highlighting the expansion and contraction phases with the standard deviation of each period. It is important to note that the growth and accelerated economic development phases are several times longer than the economic stagnation phases when Azerbaijan's economy rebooted for a new phase of progress and development.

Table 1: Production Variables analysis result

No.	Turning points (year)	Duration (years)
1	2002	6
2	2008	6
3	2014	6
4	2020	6
5*	2026*	*
6	2005	6
7	2009	6
8	2015	6
9	2021	6
10	2027	6*

Source: Author

Specification and verification: The canonical or the Cobb–Douglas multiplicative-exponential production function of the following equation (8) is the most popular:

$$Y = a_0 * K^{a1} * L^{a2} \quad (8)$$

where,

Y is the Gross Domestic Product (GDP) of the country, K represents fixed assets, L labor resources, the number of employed people in the country, a_0 is a coefficient that takes into account the influence of factors not included in this equation, its specific numerical value, for example, a constant that takes into account the possibility of innovative acceleration of the country's development or progress in science and technology,

a_1 and a_2 are the coefficients characterizing the corresponding rate of GDP growth with the production variables.

Considering the features and specifics, the described assumptions and approaches are applied to model the dynamics of Azerbaijan's GDP. As a result, the three-factor production function (PF) seems to be more relevant to the economic reality, as it determines the relationship between the GDP output (y) and the main factors of production: investments I, labor resources L, and scientific and technical factor, and which changes over time t:

$$y = f(I, L; t) \quad (9)$$

Tinbergen (1942) introduced science and technology development as an independent factor in the production function. This introduction conceptually improvised the model of PF (8) known as the Cobb–Douglas production function and can be explained in the the following equation:

$$Y = a_{0*} e^{\lambda t} * I^{a1} * L^{a2} \quad (10)$$

Where e is the base of the natural logarithm, λ is the coefficient that determines the calculated rate of technological growth for a specific period t.

The production function (10) was estimated basedon the initial data provided in Appendix 1. By taking the logarithms of the left and right sides of equation (10), simplified by changing variables, and analysed by the nonlinear least squares method using R, a statistical tool (Shipunov & Baldin, 2014), after two iterations, the following Model A is obtained:

$$Y = 0.0053 * e^{0.039 \times t} * I^{0.646} * L^{2.898} \quad (11)$$

While developing the model (11), the following values were obtained from Student's t-test for the coefficients I and L: 0.086; 0.923; with the corresponding p-value of the model equal to 5.572e-14, which is less than 0.05. In addition, the average approximation error is quite significant. The parameter for L is much greater than 1, i.e., ambiguously overrated. According to Greene (2008) and the econometric parameters provided in Table 2, Model A needed further improvement. To do so, an additional variable that characterises the workforce—the nominal average wages has been considered.

A modified Cobb–Douglas–Tinbergen function model was used to reflect the statistical dependence of output on investments in fixed assets and labor, taking into account the pace of development in science and technology, and estimated (modeled) the production function for the GDP in Azerbaijan. By considering all the variables discussed above, the following Model B is obtained:

$$Y = 0.0426 * I^{0.107} * L^{0.454} * Z^{0.749} * e^{0.039*t} \quad (12)$$

and Model C

$$Y = 0{,}035 * I^{0.131} * L^{0.371} * Z^{0.666} * e^{0.068*t} \quad (13)$$

where Z is the nominal average wage.

The econometric parameters of Model B and C improved. The average percentage of deviation in the model parameters improved by almost three times and equaled 6.6%.

In Model B, the elastic coefficients are greater than unity, corresponding to increasing returns to scale, i.e., a one-percent increase in the cost of factors increases the result by more than one percent. Technological advancement has been found to have a significant impact on the profitability of the economic system. Therefore, there are conditions for further intensive development of the economy. In addition, the proposed estimates of Model B, according to Hypothesis 2, do not imply any substitution between investments and labor resources, and evidence of complementary factors: investments and development in science and technology are found.

Table 2: Econometric Parameters of Model

Econometric Parameters of the model	Model A (11)	Model B (12)	Model C (13)
Coefficient of multiple determination R^2	0.9597	0.9956	0.9925
Improved coefficient of determination R^2	0.9555	0.9946	0.9902
F-statistics	226.5	963.5	429.9
p – value:	5.572e-14	2.2e-16	1.152e-13
DW	0.5703	2.06	2.06
p – value:	9.944e-0.7	0.2	0.3625
BP test	8.0807	4.5329	4.3356
	df=2	df=4	df=4
p – value:	0.0176	0.3387	0.3625
Skewness	0.2037	0.1057	0.1767
p – value:	0.6518	0.7641	0.6742
Kurtosis	0.3036	0.3107	0.5792
p – value:	0.5816	0.5773	0.5466
AIC	-93.6	-138.3	-111.1
BIC	-89.2	-131.8	-105.8
N	22	22	18
Residual standard error	0.0259	0.0090	0.009932
MAPE	18.2%	6.6%	6.8%

Source: Author

Conclusion

The paper discusses the results of two and three-factor models developed based on Cobb–Douglas production function with different econometrics, variables, and the development rate in science and technology. With a probability of 99%, the considered models were determined as adequate to the source data with a high coefficient of determination. For result validation, functional variables were considered and defined as the turning points in the development of qualitative indicators like average labor productivity, average labor intensity, average capital intensity, and average capital productivity of the production factors in the economy of Azerbaijan. The variables of these indicators were identified as cyclical.

The turning points coincide chronologically with the boom and recession periods in the economy of Azerbaijan, identified as both short-term and medium-term economic cycles (Akhmedova, 2018).

An analysis of production functions allows identification of the intensive and extensive types of economic systems and a country's economic growth. According to Models B and C, the sum of the coefficients of elasticity in both models is more than 1, indicating that the rate of increase in the output is more than the rate of increase in the input variables. Therefore, there exists a positive effect of increase in production on economic growth.

Model B explains that the GDP of Azerbaijan is developing extensively, $a_1 = 0.107$ is less than $a_2 = 0.454$, i.e., an increase in labor resources by 1% leads to a greater increase in the gross domestic product than an increase in investment by the same number. The rate of development in science and technology increment

λ shows us that due to the progress in science and technology production's growth rate on average accounts for 0.039% per year for the period under study in relation to the GDP of Azerbaijan. In this case, there is a small impact of technological development on the profitability of the economic system, as compared to other factors of production i.e. there are conditions for further intensive economic development. However, the results of Model B do not imply a substitution between labor and investments, labor, and the contribution of the progress in science and technology. In addition, the study assumes certain complementary factors of production like the number of people employed in the economy, and the nominal average wage, and investments in fixed assets and progress in science and technology.

Calculations demonstrated a similar pattern in Model C: the GDP of Azerbaijan is developing extensively, a_1=0.131 is less than a_2=0.371, i.e. an increase in labor resources by 1% causes a greater increase in gross domestic production than an increase in investment in fixed assets by the same number. In addition, model C indicates that science and technological development in Azerbaijan contributed to its GDP growth by 0.068%. The results of Model C do not imply substitution between labor resources, investments, and the contribution of progress in science and technology, but imply interchangeability of the number of employed people, their wages, and investments in fixed assets. As is shown by the analysis of the considered models, among other factors of production, progress in science and technology, which also includes new information and the knowledge economy, still has little effect on the GDP of Azerbaijan. This indicates a hidden potential, which should be taken out of the shadows. It is believed that the unutilized potential of production factors requires additional study and is in fact, a fount of unexplored opportunities.

The present study on production variables helped in identifying the turning points in the economic development of Azerbaijan, as well as the medium-term cycles, phases, periods of expansion and contraction of the economy growth, which alternate at regular intervals. Additionally, the influence of production factors on output has also been estimated.

The findings offered a sound base for predicting the turning points in the economic development of Azerbaijan, like the ability to predict when the economy will expand or contract. This information will help in formulating fiscal policy, planning and regulating the financial policy of medium and large enterprises. Therefore, the study findings are highly relevant and of great importance in monitoring the economic development of Azerbaijan as a whole and as a tool for forecasting and planning.

References

Akhmedova, E.M. (2018). Identification of economic cycles in the dynamic development of the Republic of Azerbaijan. Statistika xəbərləri: elmi-praktik jurnal, Azərbaycan Respublikasının Dövlət Statistika Komitəsi , Bakı ş., s. 108, səh. 59-66.

Azerbaijan State Statistics Committee (2021) Industry of Azerbaijan, Statistical yearbook. Retrieved from: https://www.stat.gov.az/source/industry/

Bessonov, V. A.& Tsukhlo, S.V. (2002). Analysis of the Economic Dynamics of the Russian Transition Economy. Moscow: Institute for the Economy in Transition.

Cobb, G.W.,& Douglas, P.H. (1928) A theory of production. // Amer. Econ. Rev. - P. 139-165.

Diewert, W. (1971). "An Application of the Sheppard Duality Theorem: A Generalized Leontief Production Function", The Journal of Political Economy, Vol.79, № 3

Green, W. G. (2016). Econometric Analysis. Book 1, translated from English; Sci. Ed. S.S. Sinelnikova and M.Yu. Turuntseva. Moscow: Delo publishing house, RANEPA, 760 p. (Academic textbook). 7th edition.

Greene, W. H. (2008). Green Econometric Analysis A guide to modern econometrics / Marno Verbeek. – 4th ed. John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex PO19 8SQ, England

Griliches, Z. & Mairesse, J. (1995). Production functions: the search for identification // National Bureau of Economic Research, Working Paper No. 5067

Həsənli, Y. (1998). *"CES-istehsal funksiyası və onun qiymətləndirilməsi,"* Az.EA-nın xəbərləri, Fizika-texnika və riyaziyyat elmləri seriyası, № 6, Bakı, s. 57-60

Plakunov, M. K. & Rajackas, P. Ji. (1984). Production functions in economic analysis. Vilnius: Mintis, 308 p.

Shipunov, A. B. & Baldin, E. M. (2014). Visual Statistics. We use R! Moscow: DMK Press, pp. 173–190.

Tinbergen, J. (1942). Professor Douglas' Production Function. Revue de l'Institut International de Statistique / Review of the International Statistical Institute, 10(1/2), 37–48. https://doi.org/10.2307/1401184

Tsvetkov, V. A. & Zoidov, K. Kh. (2010). Study of Economic Cycles in the Countries of the Post-Soviet Region, Moscow: TSEMI of RAS, 310 p.

Zellner, A., Kmenta, J., & Dreze, J. (1966). "Specification and Estimation of the Cobb-Douglas Production Function Models", Econometrica Vol.34, № 4

Zellner, A., & Revankar, N. (1969). "Generalized Production Functions", The Review of Economic Studies, Vol.36, № 2