

USAGE OF 3D TECHNOLOGIES IN STEREOMETRY TRAINING

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Abstract: StereoMV is a part of a dissertation on a topic "Stereoscopic Training System". The main purpose of the stereo system is to facilitate learning of the concepts of stereometry. The system is the author's and offers a new approach for generating geometric bodies. The boundary method is used for creating the figures, which is based on the method of the boundaries of Isaac Newton and the "Indivisible" of the Italian mathematician Bonaventura Cavalieri. The study shows how by using 3D technologies such as virtual reality and 3D printing, the process of teaching in stereometry classes will be significantly facilitated. The report presents a comparative analysis of the existing and new way of creating a regular polygon. The graphics can be presented in two ways, namely: solid and transparent. The stereo system provides an opportunity for converting from solid to transparent. The 3D model depends on its geometry and appearance. Geometry means mathematical description and the appearance is defined by the: material, texture, light and transparency. After creating the stereometric figure, the system enables it to be converted to .obj file (mesh).

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Introduction

The building of profound knowledge is faster through the so-called digital model which can be realized by the appearance of information technology and virtual reality devices. It is now possible to use this technology in mathematics training. This necessity results from the fact that when studying stereometric figures, problems arise in understanding the objects. In a case when the real object does not coincide with its projection on a plane. The usage of the modern information technologies as Virtual reality, Augmented reality and 3D printing allows the problems of this type of thinking to be overcome. StereoMV is a part of the dissertation work with the topic Stereoscopic Training System. It represents a new technology in the stereometry training and facilitates the studying of the solid figures as: prism, pyramid, cone, sphere etc. It makes terms connected with them easier to understand, as for example: vertex, edge, face and section. One of the most important characteristics is that it can make the invisible – visible and inaccessible – accessible, as at the same time it will give a real idea to the learner in the process of studying of the geometric figures, including the geometric solids as well. Existing regular polygons are currently characterized by the number of vertices and radius (Petkov, 2013).

Aim/Purpose: The aim of the present work is to represent a new stereoscopic application for the studying of geometric figures from the field of stereometry. It shows how with the help of the virtual reality technologies and 3D printing the training process in the mathematics class is facilitated. A new boundary method is presented for generating a regular polygon and truncated pyramid on the basis of existing methods.

Literature Review

The Usage of 3D modeling in school

3D is everywhere around us and this type of graphics is applied in advertising, the film industry and the computer game design. It is necessary for 3D modeling to be introduced in education as well. During the process of studying stereometry, spatial thinking is important. Also, of particular importance is the transition from a spatial image of a real object to a graphic image i.e. in other words, from a three-dimensional image to two-dimensional.

Not only professionals but beginners are engaged in 3D modeling:

- Development of the student's cognitive activity
- Creative work
- Spatial thinking
- Interest in stereometry

The students need to learn how to create 3D models. By these types of programs, the student can now see complex three-dimensional figures that are difficult to be mentally visualized. The teacher must show the capabilities of modern softwares for processing computer graphics. Already, except in

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Bulgarian schools, there are also documented results of the use of software in stereometry in other European countries.

- The students were given an application to install on their computers and when it starts they look at three-dimensional geometric models.
- Students solve the problems using all their knowledge of stereometry on a piece of paper.
- At the end, they analyze the results obtained, which are new methods and organization of the training in stereometry.

There are documented results from the use of virtual and augmented at school (Aleksandrovich, 2019). Playing activity is a leading activity in school because learning at this age is connected with playing games. For the realization of playing techniques in the learning process, it is important to use teaching material as a means of play. Augmented reality - is a new method of access to information (Aleksandrovich, 2018). It is a combination of: the real objects around the person and the virtual reality that are computer generated. There are many types of software that help to use augmented reality for receiving the necessary information about the environment. The modern smartphones and tablets allow this. The use of such applications enables the transition from traditional knowledge techniques to the newer ones. This would stimulate the development of imagination and spatial thinking, as well as the understanding of the perspective. The use of Information and Communications Technology (ICT) in school should correspond to the age of the students (Robert, 2014)

To introduce 3D modeling and augmented reality at school, students must rely on the following goals:

- Playing activity at school through usage of 3D modeling
- Children should choose which technology to use when working with projects
- A motivation for children and their parents is created in a process of work with the 3D technologies
- Children learn to be independent

3D Printing

It is known as a supplementary production technology. In this way, 3D printing is far from the mass production line and is close to the single personalization. First, the virtual design of the object is made. This kind of design can be made with the help of a program of the three-dimensional modelling. The technology of the three-dimensional scene is applied in different and various sectors.

Advantages:

- Lower prices
- Save time
- In 3D models different materials can be used

The three-dimensional printing can be applied in almost all categories. If the last industrial revolution brought us mass character of the production in factories, then the digital 3D printing can return the mass production back to the individual craft skills in home (Gebhardt & Fateri, 2013).

Unfortunately, not all file formats can be used for 3D printing. The most used of them are: STL, OBJ, AMF and 3MF.

Not all 3D file formats are formats for 3D printer. The .OBJ file format is suitable for the colour printer, it can keep information for the object in the file format .MTL.

These files can define ambient colour, diffuse colour, specular colour, transparency.

One vertex is defined in the beginning of the row with a letter v and the corresponding coordinates. Colour values are from 0 to 1, the material describes visually the polygons and the information is kept in a file with an extension .mtl. One such file can keep information for more than one material.

- Ambient – Ka
- Diffuse – Kd
- Specular – Ks

The file with an extension .obj represents 3D geometry of the model and holds information connected with:

- Every vertex
- UV position of the texture coordinates of every vertex
- Normal

- Information of the faces defined from a list of vertices (Gordon, 2019).

Computer modeling via Java

The model depends on its geometry and appearance. Through geometry the mathematical description of the 3D model is given, while the appearance is defined by the colour, texture and material (Chung & Cheng, 2009). The geometry can consist of a number of triangles. These type of objects have priority over the others. The class in Java with the help of which the geometry and the appearance is defined, is Shape3D (Zhang & Liang, 2007). For the creation of the edged body with the help of the class mentioned above, the same can be printed after it is converted into a file with an extension obj.

The 3D content can be classified into three categories: geometry information, appearance information and scene information. The geometry of the model is stored as a data set of 3D points. The surface of the model is then saved as a series of polygons. The design of the three-dimensional model only with points and faces, is not very convenient. The CSG Paradigm 3D forms are built of Boolean operations on primitives of simple shapes such as: cube, cylinder, and sphere. The material includes applying a texture to the surface of the model. This is achieved by mapping of each three-dimensional vertex to the corresponding point in the two-dimensional image. A model that does this, should store these texture coordinates in a 3D data file. For the visual representation of the material, an important role play surface, light and surroundings properties. One surface may have components such as: ambient, diffuse and specular that show colour, light and intensity. Also the material can be transparent and being define one after another in a file with extension .mtl (McHenry, 31.10.2008).

The term "scene" means the layout of the model regarding the camera, light and other three-dimensional models. More than 100 file formats are known. At the same time, there are many types of software that can export and import, in other words to convert different 3D files. The .obj file format is based on the text open file format, developed from Wavefront Technologies. This format is accepted from other suppliers of 3D graphic applications. The object file format supports: the geometry in the form of vertices, faces and parametric surfaces, vertex, normals, textures, material.

Methodology

3D geometric models

When more complex geometric models need to be created, special software products are used. There are different file extensions, from which the most commonly used are the files with the .obj extension. Data on the construction of these objects can be viewed by a text editor such as notepad. In this type of data, the normal vectors "vn", texture coordinates "vt", the vertices "v" and for each of the faces "f" are described.

A large part of the three-dimensional models can be represented by tables, respectively for the vertices, edges and faces. This can be done within the range of the application software. However, when the objects are more complex to create, specialized programs such as Art of illusion and Blender can help. These programs should use appropriate formats for representing the geometric models. The most used of which are: .obj, .3ds, vrml, .x3d etc. In this case, Java3D can work with all of them. The data about the models is kept in electronic - digital format and by using text editor like notepad, it can be viewed and subsequently modified. When reviewing the data, the following symbols can be found such as: "v", "vn", "vt" and "f", respectively for the vertices, the normal vector, texture coordinates and face coordinates.

There are three methods for creating three-dimensional models:

- Polygons - points from the three-dimensional space called vertices connect to form a polygon mesh (Ragnemalm, 2010).
- Curve modeling – the surfaces are defined by curves defined by number of control points (Zhang & Liang, 2007)
- Digital sculpting – a new method for generating of three-dimensional models by 3D modeling software such as: Blender and Art of Illusion

For the realization of the stereo system, the first two methods for creating the objects are used.

Spline curves are a very important tool in computer graphics. The B-Spline curve is a smooth curve which is defined by sequence of control points. The B-Spline curve can be converted into Bezier curve. Java2D and Java3D offer primitives that form a hierarchy of classes. Of course, the

programmer can create his own primitives. One spline curve consists of a sequence of polynomial curves that are mainly used in CAD.

The Bezier surface is formed by extending the Bezier curve to the surface relatively to another Bezier curve and the method used is called the tensor product. The most used Bezier surface is the bicubic surface defined by 16 control points (Zhang & Liang, 2007).

Solid and wiframe graphics

The 3D graphics is positioned in the three-dimensional space (Terrazas, Barlow, & Ostuni, 2002). It can be simpler or more complex. The geometric model is a main component of this type of graphics. It is divided into three types, respectively: Sceleton, Surface, Volume graphics (Maleshkov, 2014). Here, the objects characteristics are set, as well as information about creating them. The most important information about the model is determined by the space and how the primitives are involved in the construction of the object.

The information of their representation also is contained. For developing of the stereoscopic system of training in strereometry, exactly these models for presenting the objects are used. They are interconnected, as the surface models are considered to be a continuation of the skeleton models. In the first model, the necessary information for constructing the graphics contains information about vertices and edges. In the second type, bounding objects surfaces are used. Here appears information about the faces of the model.

Types of solid/wireframe graphics

- Many 3D graphics software use wireframe to represent the objects. These type of graphics consists of a series of vertices and lines connecting these vertices and forming the frame of the object (Palmer, 2001). In the graphics of solid type, the objects are represented by a higher solidity. They are surface and voluminous. There are algorithms that can be used to convert from wireframe to solid. The process is called conversion from a wireframe to a solid body (solidifying wireframes) (Srinivasan V., 2005)
- The basic solids models are: primitive instancing, sweep presentation, boundary representation, and Constructive Solid Geometry. In the first model, the modeling system is defined by a series of solid-shaped primitives. In the second model, the technique sweep is used (Chen & Chen, 2008). In b-reps model, the object is described in terms of its boundaries such as vertices, edges, and faces. This type of model in the stereo system is, for example, the bodies pyramid and prism, described by edges and faces. CSG has a combination of primitives for creating complex 3D objects (Abdul, 2008).
- Solid modeling uses topological information, in addition to geometric, to represent objects completely. The advantage of solid models over others is that the solid model results in a more precise design. The disadvantage is that there is information that cannot be seen.

Mathematics methods

In the stereoscopic application a new boundary method is used, created by the author of the stereo system. This approach is on the basis of the Newton's boundary method and the Method of Indivisibles of the Italian mathematician Cavalieri. The presented method takes part in the calculation of the values of the vertices in the three-dimensional space. The aim is the creation of the regular polygon which is characterized by the number of vertices and the length of the edge. The polygons created through it are much easier for manipulation and processing. For now, it is limited to 6 vertices. By means of the new approach the figure of a truncated pyramid can be generated.

The boundary method is outlined by Newton in twelve lemmas (Nyuton, 1989). The listed Newton lemmas are of great importance. He was the first to introduce this concept. Subsequently, the method was further developed by other mathematicians. With the help of the boundary method, the cylinder volume can be calculated. The volume of a cylinder is called the limit to which the series of volumes, of the inscribed in it regular prisms, tend to, when the number of their vertices grows unlimitedly (Ivanov, 1965).

The formula of the volume of a pyramid can be drawn up by the Cavalieri's Indivisibles Method. The pyramid height is divided into a certain number of parts, after that sections are made that are parallel to the base. The relationship between the face of the base and its parallel section is used.

Results

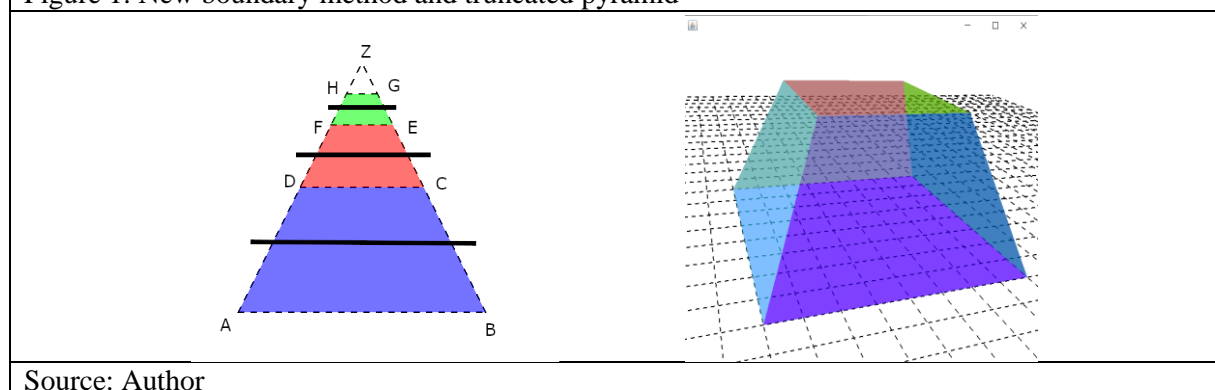
In order to explain what the boundary is, it is the most appropriate to describe it through a mathematical problem, and the most suitable is one which is geometric. Now we are going to explain the new boundary method.

We have an isosceles triangle ABZ, and its middle line segment is constructed Figure №1, after that the trapezium face is searched in the colour blue. In the triangle DCZ, the middle line segment is built, and the trapezium face is searched in the colour red. These repetitive actions are done to infinity, and the following pattern of decreasing sequence is observed.

Let us have an isosceles triangle of which the length of the base and the height of it are equal to one, the following row of numbers is observed: $\frac{3}{8}, \frac{3}{32}, \frac{3}{128}, \frac{3}{512} \dots$, with the following pattern: $\frac{3}{4^{n+m}}$, where m is const and n number $n \in [0, +\infty)$. In this triangle, countless many proportional sections can be formed, which are constructed by an average segment in triangle and trapezoid. The solid black line denotes the middle section of the trapezoid.

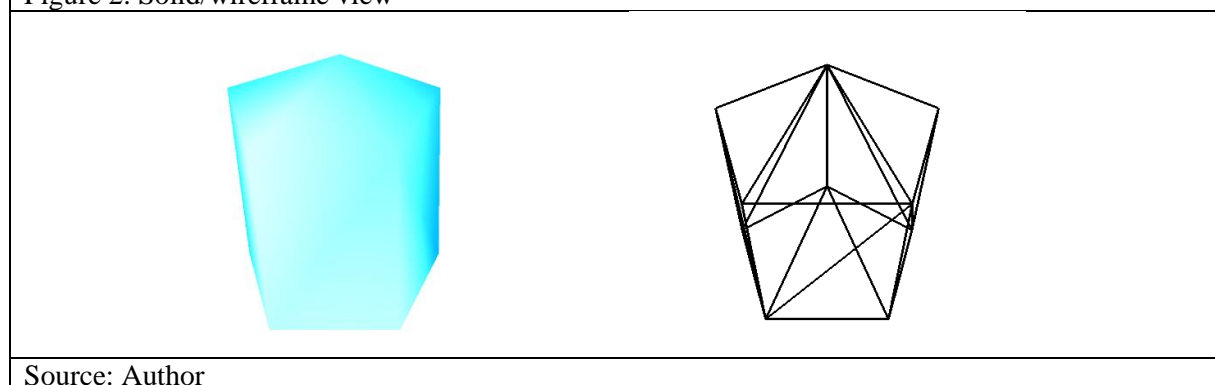
Newton explains the notion of boundary though physico-mathematics (Bashmakova, 1975). The boundary method is also known as the method of the first and last relations and also as the method of appearing and vanishing quantities. The concept “last velocity” must be understood to mean not the velocity with which the body moves until its arrival at the last place, when the movement is terminated, but the velocity that the body has reached at the very moment when it has arrived at the last place. Similarly, the last relation of vanishing quantities must be understood as the relation between the quantities not before or after their disappearance, but the relation with which they disappear. In other words, in order to reach the boundary, the quantities must be reduced to infinity.

Figure 1: New boundary method and truncated pyramid



Source: Author





Figure 2: Solid/wireframe view



Source: Author

Table №2 and Figure №2 present the geometric information, that participates in the computer generation of a regular pentagonal prism. A new boundary method created by the author of the system is used to calculate the vertices of the prism. The user selects, from the program interface the type of base of the prism, which is a polygon. After that, the length of the main edge and the height of the body are entered. At the same time, if the values on the abscissa or the applicate is a number other than zero, then the final result will be an inclined prism.

In the study, the polygon that forms the basis of the prism and pyramid is limited to six vertices. It is characterized by the number of vertices and the length of the main edge. It is an innovative way of creating it, and in many ways, it is superior to the already existing one, which is based on trigonometry, depending on the number of vertices and the radius.

Polygon	New polygon Value a	Old polygon Value a	Value R new polygon	Value R old polygon
	1	1	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
	1	$\frac{\sqrt{2}}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$
	1	$\frac{2 * \sin 36}{\sqrt{3}}$	$\frac{1}{2 * \sin 36}$	$\frac{1}{\sqrt{3}}$
	1	$\frac{1}{\sqrt{3}}$	1	$\frac{1}{\sqrt{3}}$

Source: Author

Number of vertices	X	Y	Z	# Material mtllib root.mtl # Vertices (5) v 0.0 -177.98245 0.0 v -241.5 0.0 0.0 v -150.0 285.7057 0.0 v 150.0 285.7057 0.0 v 241.5 0.0 0.0 # Textures Coordinates (4) vt 1.0 0.0 vt 1.0 1.0 vt 0.0 0.0 vt 0.0 1.0 #End Texture Coordinates # Faces (3) usemtl root f 1/4 2/3 3/1 f 1/4 3/1 4/2 f 1/4 4/3 5/1 # End Faces
1	-a/2	a/2*b	0	
2	a/2	a/2*b	0	
3	a/2*c	0	0	
4	0	-a/2*d	0	
5	-a/2*c	0	0	

Source: Author

Table 1 compares the new polygons, based on the new boundary method, and those created according to trigonometry rules. With new polygons, the length of the side remains the same, while the radius gradually increases. In the case of the existing polygons, the length of the side is gradually decreasing, while its radius remains the same. Initially, the length of the polygon side is equal to 1.

Table 2 presents the parametric values at the vertices of a regular pentagon where b, c and d are auxiliary variables. Next to it, the corresponding .obj file is presented.

float b = (float) Math.sqrt(3.6279); float c = (float) 1.61; float d = (float) Math.sqrt(1.4079);

.Obj is a definition of the geometric file format of Wavefront Technologies. It presents the 3D geometry of the models and provides information about:

- Geometric vertices
- Texture coordinates
- Vertex normal
- And the polygon faces

Where the vertices are presented counterclockwise or clockwise.

What the objects look like depends on the colour, texture and light. In terms of colour and light, they have common characteristics such as ambient, diffuse and specular, RGB colours. This type of information is contained in a file with the extension .mtl, and could be modified manually with a program such as notepad. The system enables conversion from Shape3D to Mesh, with .obj file extension. For this aim, in this article, it is used the JavaFX library - FXyzlib of Jose Pereda.

Conclusion

The study shows how today, with the help of 3D technologies, such as virtual reality and 3D printing, the process of studying objects in the field of stereometry will be facilitated for students. The gaming activity is a leading activity in school. The purpose of the software is precisely to make learning a game. The report proposes a new approach that was used to create the stereo system. It represents a boundary method that participates in the generation of geometric figures. It is based on the boundary method, the method of indivisibles, and Thales' theorem. It is currently limited to creating a regular polygon of up to 6 vertices. A comparison is also made between new polygons and existing ones based on trigonometry. The presentation of three-dimensional models can be solid or wireframe. The future work will be to write a Java node in a .obj file with Java3D library. The future work will continue until reaching of the boundary of an inscribed regular polygon in a circle.

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