

PRACTICAL PRODUCTIVE ACTIVITIES FOR THE DEVELOPMENT OF MATHEMATICAL COMPETENCES

Maria Temnikova¹

Abstract: Development of knowledge, skills and competences for the mathematical quantity of “length” and its measurement units represent important part of the overall education in mathematics for students from Grades 1. – 4.

This research work systematizes some theoretical concepts related to the competences. Based on the analysis of the relevant legislation in force in the Republic of Bulgaria researchers presented the relations between the specific mathematical competences and terminology from the competency Cluster “Measurement”.

The research work is aimed at the development of such a methodology system of work which gives priority to practical productive activities thus facilitating development in primary school students of a higher level of knowledge, skills and competences related to the mathematical quantity of “length” and its measurement units.

The new system of work was applied during the compulsory classes in mathematics in Grades 1. – 4. The research work presents some of the developed and applied options of practical activity.

The researchers studied the objectiveness, the validity and the reliability of the set of tools and means used for doing diagnostics as well as the difficulty and the dividing (separating) strength of the mathematical tasks included in it. The results from the experimental work were statistically processed.

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Introduction

Over the recent years the mathematical tests in the Republic of Bulgaria for national external assessment for Grade 4 show that students have results lower than expected when solving tasks related to the mathematical quantity “length” and its measurement units.

This required detailed studying of some of the specifics of the methodology of work with the mathematical quantity “length” and its measurement units in Primary school.

Over the last decades the Competency approach became widely used in the countries of the European Union, the United States, Russia, etc. and the terminology “competency” and “competences” were in its foundations.

As a result of the analysis of foreign and Bulgarian pedagogy literature it was found that there are a number of studies related to issues concerning “competences”.

Common for all such studies is the understanding that “competences” represents a person’s ability to manage different tasks. According to Khoutorskoy, a competence is “an alienated upfront raised social requirement (norm) to the educational preparation of the student, necessary for his efficient productive activity in given sphere” (Khoutorskoy, 2017, p. 86). For Neminska competences are “specific values, attitude, skills, knowledge and understanding which get mobilized and deployed for the purposes of regulating competent behavior” (Neminska, 2018, p. 80).

Accent of the research work was put on development of the first two groups of competences - learning to know and learning to do. These two groups represent part of the four main groups of competences determined by Jacque Delores (Delores, 1996, p. 37) which are related to the changes in the area of education in Europe.

Khoutorskoy (2011) developed a hierarchy of competences where he arranged them in three levels: key competences, general subject competences and subject competences. He refers to the last group of competences as “particular case to the two precedent levels of competences which can be developed within the frame of the educational subjects”.

The aim of this research is to systematize and present the main theoretical concepts regarding specific competences and consequently to develop a new methodology system of work incorporating multiple practical activities as well as to study the level of development of knowledge, skills and competences related to the mathematical quantity “length” and its measurement units in the education of mathematics for Grades 1.-4.

¹ Trakia University, Faculty of Education, Stara Zagora, Bulgaria, mpt66@abv.bg

The object of the study was the process of education in mathematics in Grades 1.-4.

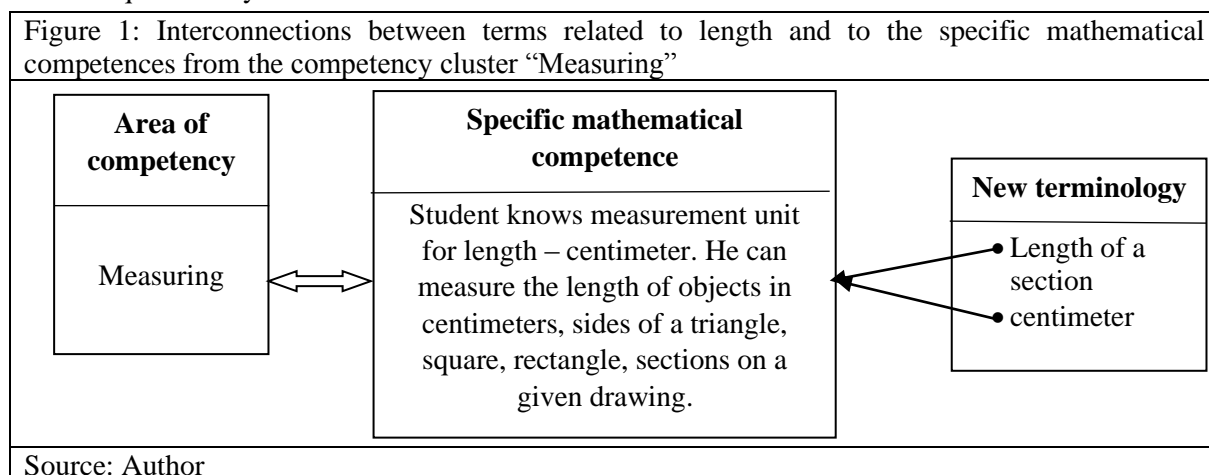
The subject of the study was the effect of the applied new methodology system of work with its multiple practical activities aiming at the development of specific competences related to the mathematical quantity “length” and its measurement units in the education in mathematics for Grades 1.-4.

In order to achieve the aim of the study the following tasks were completed: study and theoretical analysis of: research works of foreign and Bulgarian authors related to the competency approach, the competences and legal documentation concerning the education in mathematics in Primary school; the systematization of specific mathematical competences and the presentation of typical specific interrelations between themselves; the development of a methodology system of work where priority was given to practical activities as well as the application of this system in the education of mathematics for Grades 1.-4. tending to facilitate the forming of specific mathematical competences; the study of the level of knowledge and skills of the Grade 4 students related to the mathematical quantity “length” and its measurement units; a comparative analysis and assessment of the results from the empiric study, a summary; and the present conclusions.

For the purposes of the empiric study the following criteria were introduced: knowledge and skills regarding the mathematical quantity “length”; knowledge and skills regarding its measurement units – millimeter, centimeter, decimeter, meter, kilometer; knowledge and skills for operating with the same measurement units for “length”.

This hierarchy and competence levels were adopted for the purposes of the research work with special attention put on the specific subject mathematical competences. Students’ knowledge and skills regarding the quantity “length” are formed and developed during the education in mathematics for Grades 1.-4. As the intention of the research work was to develop specific mathematical competences, the author had to systematize them based on the relevant legal documentation in force in the Republic of Bulgaria. The study offered detailed analysis of: the requirements in respect of the educational content in mathematics for primary school and educational programs in mathematics for Grades 1.-4. Based on this analysis, specific interrelations were presented. Figure 1 demonstrates part of the interrelations between the specific mathematical competences and basic terminology related to the mathematical quantity “length”.

The term “mathematical quantity” is a fundamental one and is used not only in mathematics. H. Roegiers (1985, p.133) considers mathematical quantities as characteristics of objects or events that could be quantitatively determined



During the analysis of the educational content in mathematics for Grades 1.-4. It was found that the material for quantities and actions with named numbers is not studied individually but is divided into classes and topics in relation to the studied arithmetic and geometric material.

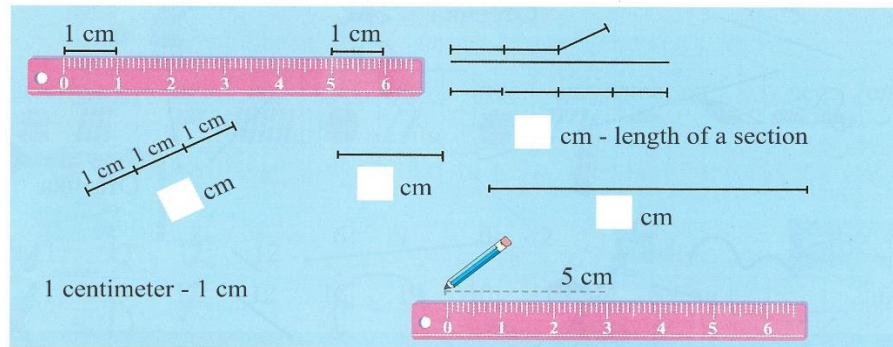
From the mathematical quantities studied in the Primary school, the biggest weight is put on the quantity “length” and its measurement units that are based on a decimal numeric system.

The term “length” is getting introduced still in kindergarten where relative measurement units are used: pace, hand palm, tape. For his reason the initial practical work commences with measuring

different distances in the classroom which have been determined by the teacher in advance. For the purpose paces and hand palms are used as well as game-oriented methods of measurement. Further the works can proceed with practical activities related to measuring certain lengths (like paths, bridges, etc.) using tape and involving some entertaining elements in parallel. These activities were developed on the working sheets of the students.

According to the current educational programs in the republic of Bulgaria the Grade 1 students learn the term “length of a section”. First the measurement unit centimeter is introduced as they can directly work and measure lengths with it. This introduction is done through demonstration and practical examples. For the purposes of solving relevant mathematical tasks of the below type, the teacher and the students make use of a ruler (Bogdanova and Temnikova, 2016, p. 52):

Figure 2: Task for the introduction of the centimeter as a measurement unit for length

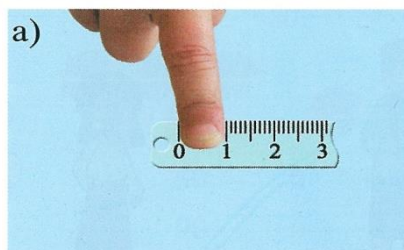


Source: Bogdanova et al., 2016

As a demonstration and visualization tool for the purposes of developing general understanding about the mathematical quantity “length” and its measurement unit “centimeter” use the following task (Bogdanova et al., 2016, p. 20):

Figure 3: Task for visualization of the length of a centimeter

You also show on your ruler:



Source: Bogdanova et al., 2017

The next productive practical activities are related to measuring the length of a dotted line and the sides of geometry figures.

In Grade 2, in parallel with introducing the numbers up to 100, the measurement unit decimeter and meter are introduced. Practical use of the measurement unit decimeter is related for example to measuring the length of a child’s palm.

Decimeter is determined as a measurement unit equal to 10 centimeters: $1 \text{ dm} = 10 \text{ cm}$. Accent is put on the decimal ratio between neighboring measurement units for length. The practical work which is assigned to the students is related to measuring the length of a textbook, notebook, pencil, etc.

Motivated introduction of the measurement unit “meter” is based on the following visualizing and operating tool as shown on Figure 4 below. (Bogdanova et al., 2017, p. 21)

It is necessary to demonstrate different models of meter: tailor’s meter, carpenter’s meter, measuring tape, etc.

The students need to be included in practical tasks related to measuring different distances in the classroom: length of a working table, teacher’s desk, blackboard, etc.: (Bogdanova et al., 2017, p. 21)

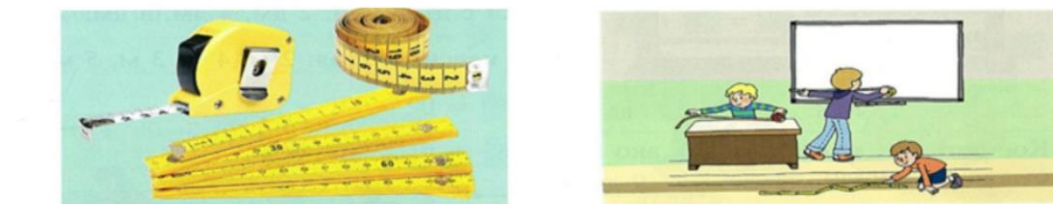
Figure 4: Task for visualization of the length of a meter



Source: Bogdanova et al., 2017

Figure 5: Task for practical measuring in meters

With the help of these tools one can measure length of object



- a) Measure in the classroom or in the school yard lengths of: 5m; 3m; 7m; 2m.
 b) Compose tasks for the measured lengths and solve them.

Source: Bogdanova et al., 2017

The students are organized into teams. During mathematics classes, students also measure the length of prepared in advance ribbons, tapes, box edges, etc. The practical activities related to measuring continue with measuring distances in the school yard that have been specified by the teacher in advance: the length of a football playground, of a basketball playground, distance between trees, etc. The students work in teams of two and record the results of the measurements.

The following ratio between the learned measurement units are introduced: $1\text{ m} = 100\text{ cm}$ and $1\text{ m} = 10\text{ dm}$.

The teacher and the students discuss where practical measuring with a meter can be applied: for measuring material for clothes, the height of a room, trees and length of small distances.

Following the introduction of the numbers up to 1000 in Grade 3 the students get familiar with the measurement unit millimeter and kilometer. The motivated introduction of millimeter is related to the measuring of very small distances. Thickness of a child's nail is 1 mm. Through a multimedia presentation the teacher demonstrates examples of small distances: needle, bolt, sides of the geometry figure triangle, rectangle and square. The teacher uses a centimeter ruler for visualization and shows that $1\text{ cm} = 10\text{ mm}$. It is very important that students see 1 mm in all divisions of their centimeter rulers as well (Bogdanova et al., 2018, p. 44).

Figure 6: Introduction of millimeter as measurement unit for length

Verbally complete to make it correct.

$100\text{ cm} = \square\text{ m}$ $1000\text{ m} = \square\text{ km}$ $1\text{ dm} = 10\square$

On how many equal distances is divided 1 cm?
1 cm = 10 millimeters ($1\text{ cm} = 10\text{ mm}$)

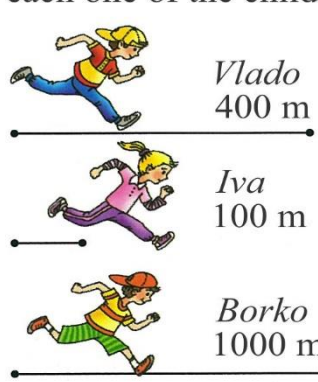
Source: Bogdanova et al., 2018

The teacher assigns the students a task to color 1 mm on a centimeter ruler drawn on their work sheets. Additionally, for solving tasks related to measuring lengths in millimeters the students are required to show precision, exactness and correct positioning of the drawing tools. The motivated introduction of

the measurement unit kilometer is related to the measuring of distances between towns and other locations. Before the respective class in mathematics the teacher must organize an excursion for the students as a form of education. The whole group shall walk 1 km in a straight line from Object X to Object Y. At Object X the teacher must attract the attention of the students to the fact that they will walk 1 km. Also, the teacher will have to explain that the distance between towns is too big to be measured using the measurement units learned so far. The motivated introduction of the new measurement unit “kilometer” follows as well as the ratio between kilometer and meter: 1 km = 1 000 m. The students solve the task (Bogdanova et al., 2017, p. 29):

Figure 7: Introduction of kilometer as measurement unit for length

How many meters run each one of the children?



Vlado
400 m

Iva
100 m

Borko
1000 m

Compare.
100 m 1000 m

Length of 1000 m is called 1 kilometer.

1 kilometer =
= 1000 meters
(1 km = 1000 m)

Source: Bogdanova et al., 2018

Because of the difficult practical implementation of distance measuring in kilometers, as a productive activity the students were assigned the task to do the following mathematical research work: to find and to put down in a table the distance between their native town and other given towns in the country. For example: Find the necessary information about the distance between the towns given in the below table and write down the results.

		Distance in kilometers (km)
	Stara Zagora – Plovdiv	_____ km
	Stara Zagora – Bourgas	_____ km
	Stara Zagora – Sofia	_____ km

Source: Author

The teacher checks if the Grade 4 students have filled in the data in the table correctly. He is asking questions about the longest and the shortest distance between the towns. These tasks help the students to acquire skills for collecting and processing information from different sources. Students are creative and compose different types of text tasks based on the collected data.

During the period between years 2016 and 2019 the researcher studied the process of development of specific mathematical competences related to the mathematical quantity “length” and its measurement units in the education of mathematics for Grades 1.-4. in two classes of students with statistically equal levels of educational achievements. One of the classes was the experimental one where the new methodology system of work was applied, and the second class was the control (referent) one.

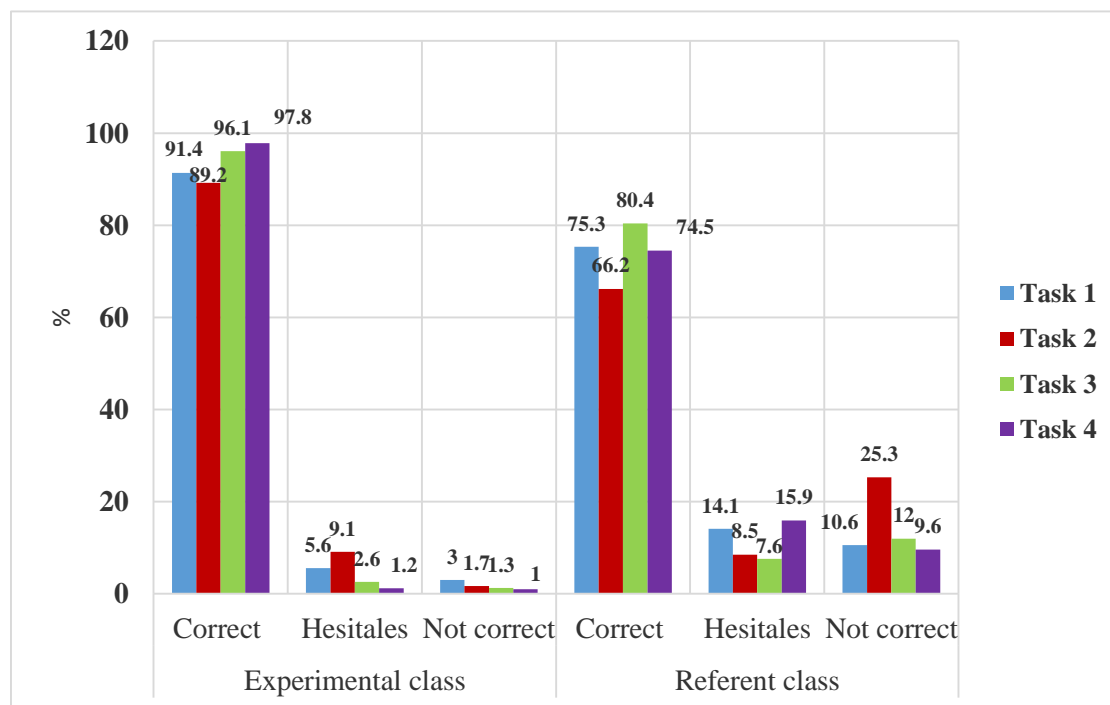
The empiric study used two types of tests: one of the tests was aimed at determining the entry diagnostic of the students’ knowledge and skills and the second one – for determining the exit diagnostic of students’ knowledge and skills.

The analysis of the test results from the entry diagnostic showed that there was no statistically significant difference in respect to the knowledge and the skills of the students from the experimental and the referent class subjected to the study.

However, the results of the exit diagnostic presented on the next figure proved to be statistically significantly different.

There is significant difference between the results from the exit diagnostic achieved by the students from the experimental and the referent class. It must be noted that 10.53% of the students from the experimental class made mistakes when working with the quantity “length” and its measurement units whereas this percentage of the students from the referent class was 21.11%. The percentage of students from the experimental class who improved their level of knowledge and skills and correctly worked with the measurement units for “length” increased from 57.61% to 93.64% whereas this percentage of the students from the referent class increased from 61.82% to 81.83%. The percentage of the Grade 4 students from the experimental class who correctly solved text tasks related to the mathematical quantity “length” and its measurement units increased by 25.17%. The percentage increase for the students from the referent class was 10.91%.

Figure 8: Exit diagnostic for knowledge and skills of the students



Source: Author

Conclusions

The application in the experimental class of the new methodology system of work involving productive practical activities led to the following results: students developed higher levels of knowledge and skills related to the mathematical quantity “length” and its measurement units in comparison with the students from the referent class; students formed and developed mathematical competencies, specific and key competences; the new system of work facilitated the process of acquiring knowledge regarding the decimal numeric system and four arithmetic operations.

Formulating the specific mathematical competences for Grades 1.-4. related to the mathematical quantity “length” and its measurement units the following was achieved: clarity regarding the starting point of the methodology work of the teacher in respect to the development of concrete competences in the students; the study shows the directions which require specific mathematical activities for the development of competences and competencies in the students.

Solving mathematical tasks related to practical productive activities regarding the quantity “length” and its measurement units helps for establishing relation between studied mathematical knowledge and its practical application, students’ interest and motivation for solving mathematical tasks is increased which facilitates the development of students’ thinking and puts them in an active cognitive position.

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